Numerical Analysis of Avionic Grounding Structures with Surface PEEC Formulation

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***Abstract*—The numerical simulation of complicated grounding structures at low frequency – from about a hundred Hertz to some MHz – is required in several cases, for instance for the modelling of direct and indirect effects of lightning on aircraft. The Partial Element Equivalent Circuit (PEEC) methodology allows the mitigation of the low frequency ill conditioning problems of the canonical Method of Moments, and also allows the excitation of the structure under analysis by direct injection of an external electrical current. This paper describes the use of a surface formulation of the PEEC methodology, including the use of an accelerated iterative solver to allow the analysis of large models. (*Abstract*)**

***Index Terms*—Method of Moments, Partial Element Equivalen Circuit, Current Return Network, Lightning, Electromagnetic Compatibility.**

I. INTRODUCTION

Several situations require the analysis and the verification of grounding structures. As an example, this is the case of an aircraft, where a dedicated conductive electrical pathway, usually named Current Return Network or Almost Equipotential Electrical Network (ALEEN) has to be integrated for the return of direct and alternating currents, faults currents, lightning current, etc. In case of aircrafts made of composite materials the body cannot assure a low impedance path, and an accurate modelling and design of such grounding structures becomes very important.

The numerical simulation of ALEEN structures is a very challenging problem. An ALEEN is a large and usually very complicated structure, and it includes small pieces and contacts; we are usually interested in very low frequency (from about a hundred Hertz to some MHz), where classical Method of Moments formulation may become inaccurate due to ill-conditioning problems; capacitive and inductive mutual coupling, skin and proximity effects have to be accurately simulated in order to precisely estimate impedances and electromagnetic field generated near the structure.

This paper describes the use of a surface formulation of the Partial Element Equivalent Circuit (PEEC) methodology for the numerical modelling of grounding structures in low frequency. The PEEC methodology enforces the Electric Field Integral Equation and uses separate basis functions for current and charge unknowns, allowing a mitigation of the ill conditioning problems of the canonical Method on Moments.

The continuity equation is explicitly included in the linear system to be solved. This gives the possibility of excite the structure by injecting an electric current in a part of the structure under analysis, and collecting the same current in another part of it, without the necessity to include any other elements (for instance loops or wires). This is particularly useful in case of modelling lightning effects, or to estimate the impedance between different points of the structure.

The paper is organized as follows. Section II summarizes the surface PEEC formulation, and Section III describes the use of the ACA acceleration technique with a preconditioner to allow the analysis of large models. Section IV reports some results.

II. S-PEEC FORMULATION

The PEEC method ([1], [2]) is an electric field integral equation (EFIE) based approach which can provide an equivalent circuit in terms of the capacitive and inductive interactions between the elemental currents and charges in the discretized structure. The degrees of freedom in the PEEC formulation are the electric currents and the charges or the coloumbian potentials, quantities that are appropriate for an equivalent circuit representation.

Let us consider a conducting structure. At any point in a conductor, the EFIE in the frequency domain reads:

$$\boldsymbol{E}_0(\boldsymbol{r}) = \frac{\boldsymbol{J}(\boldsymbol{r})}{\gamma} + j\omega\frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}(\boldsymbol{r}')e^{-jk|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}dV' + \nabla\boldsymbol{\Phi}(\boldsymbol{r}) \quad (1)$$

where γ represents the electrical conductivity of conductors, $\boldsymbol{J}(\boldsymbol{r})$ represents the volumetric current density, $\boldsymbol{\Phi}(\boldsymbol{r})$ is the electric scalar potential, $\boldsymbol{E}_0(\boldsymbol{r})$ the incident electric tangential field radiated by external sources and $k = \omega/c_0$ is the wavenumber.

By exploiting the equivalence theorem, it is possible to write the EFIE on the external surface of conductors, involving the surface equivalent current and charges. In particular, by considering conducting structures (and neglecting the magnetic current contribution), we have

$$Z_s^{eff}(\boldsymbol{r})\boldsymbol{J}_s(\boldsymbol{r}) + \frac{j\omega\mu_0}{4\pi}\iint_{S_c} G_0(\boldsymbol{r},\boldsymbol{r}')\boldsymbol{J}_s(\boldsymbol{r}')ds' + \frac{1}{4\pi\varepsilon_0}\nabla\iint_{S_c} G_0(\boldsymbol{r},\boldsymbol{r}')\sigma(\boldsymbol{r}')\,ds' = \boldsymbol{E}_0(\boldsymbol{r}) \quad (2)$$

where S_c denotes all metallization surfaces, and G_0 is the free space Green's function. $Z_s^{eff}(\boldsymbol{r})$ is the effective surface impedance which relates the tangential components of the electric field vector on the conductor surface to the corresponding tangential component of the magnetic field vector

$$E_t(\boldsymbol{r})\hat{\boldsymbol{t}} = Z_s^{eff}(\boldsymbol{r})\hat{\boldsymbol{n}} \times \boldsymbol{H}(\boldsymbol{r}) \ . \qquad (3)$$

The surface impedance includes resistive losses, skin effect and internal inductance of the conductor. Its value of can be calculated with analytical formulas in case of canonical geometries, or estimated by means of a 2D numerical modelling of the transverse section, by assuming a longitudinal current flow.

In our implementation, the surface electric current density $\boldsymbol{J}_s$ is expanded on RWG basis functions [3]

$$\boldsymbol{J}_s(\boldsymbol{r}) = \sum_{n=1}^{N_c} I_n \boldsymbol{f}_n(\boldsymbol{r}), \qquad (4)$$

and the surface electric charge σ is expanded on basis functions uniform on triangles

$$\sigma(\boldsymbol{r}) = \sum_{m=1}^{M_c} Q_m q_m(\boldsymbol{r}). \qquad (5)$$

By using (4), (5) and a Galerkin discretization scheme, (2) is translated into the following linear system of equations

$$\left(\hat{\boldsymbol{Z}} + \boldsymbol{j\omega Lp}\right)\boldsymbol{I} + \boldsymbol{APQ} = \boldsymbol{V_s}, \qquad (6)$$

where

$$\hat{Z}_{km} = \int_{T_k} Z_s^{eff}(\boldsymbol{r})\boldsymbol{f}_k(\boldsymbol{r}) \cdot \boldsymbol{f}_m(\boldsymbol{r})ds$$

$$Lp_{km} = \frac{\mu_0}{4\pi}\int_{T_k} \boldsymbol{f}_k(\boldsymbol{r})ds \cdot \int_{T_m} G_0(\boldsymbol{r},\boldsymbol{r}')\boldsymbol{f}_m(\boldsymbol{r}')ds'$$

$$\boldsymbol{P_{km}} = \frac{\boldsymbol{1}}{\boldsymbol{4\pi\varepsilon_0}}\int_{T_k} \boldsymbol{q_k(r)ds}\int_{T_m} \boldsymbol{G_0(r,r')q_m(r')ds'}$$

$$\boldsymbol{V_{sk}} = \int_{T_k} \boldsymbol{E_0}\ (\boldsymbol{r}) \cdot \boldsymbol{f_k(r)ds}$$

In (6) $\boldsymbol{A}$ represents a connectivity (incidence) matrix relating the triangles – where charge unknowns are defined – to their sides – where current unknowns are defined.

The surface current and charge density are related by the continuity equation. By applying the Galerking discretization scheme also to this equation we obtain the following linear system of equation

$$-\boldsymbol{A}^t\boldsymbol{I} + j\omega\boldsymbol{Q} = \boldsymbol{I_s} \qquad (7)$$

where $\boldsymbol{I_s}$ indicates a vector containing the external currents injected into the elements of the structure. Equations (6) and (7) represent the linear system to be solved in the S-PEEC formulation. A similar linear system can be formulated also for DC analysis (ω = 0), by assuming a surface element as a reference and decreasing by one the number of charge unknowns.

III. Iterative Solution

As for all the other full wave integral equation formulations, the use of a direct method to solve the linear system of equations given by (6) and (7) can be computationally too much expensive, since their complexity scales as $O(N^3)$ in CPU time and as $O(N^2)$ in memory requirements, N being the number of unknowns. In the case of S_PEEC formulation N is given by the sum of the current and charge unknowns, and it is higher than the number of unknowns required by the classical Method of Moments (adopting only current unknowns).

In order to allow the analysis of large models, an iterative solution method of the linear system of equation (GMRES, BiCGStab, etc.) can be adopted. An iterative solver requires the evaluation of the product between the linear system matrix and a vector, for each iterative step. In case of a full matrix with order N, such product has a computational complexity and a memory requirements of $O(N^2)$. In case of large models, this can lead to unfeasible complexity and memory requirements. An acceleration technique and a preconditoner can be used to evaluate the matrix-vector products in a computationally efficient way, and to reduce the number of iterations to solve the linear system, respectively. The adopted techniques are briefly summarized in the next subsections.

A. ACA Acceleration Technique

Considering the S-PEEC linear system (6)-(7), two dense submatrix-vector products are required: $\boldsymbol{LpI}$, $\boldsymbol{PQ}$. All the other submatrix-vectors products involve diagonal or sparse matrices. The acceleration is then achieved when these two submatrix-vectors products are computed in an efficient, though accurate way, by means of an acceleration strategy. The same acceleration strategy allows the reduction of the dynamic memory required.

In the frequency range of interest, the usual structures to be analyzed are not large with respect to the wavelength, and the Green's function (the interaction kernel) is a smooth operator. For these reasons, the Adaptive Cross Approximation (ACA) allows representing the full sub-matrixes of the S-PEEC linear system in a very efficient way, with no particular behavior differences in the range of frequencies of interest. Other acceleration algorithms, like the Multilevel Fast Multipole Approach, should be adopted for high frequency band.

The ACA algorithm ([4], [5]) is based on the fact that an impedance matrix of rank m×n related to two very well separated blocks of basis functions can be approximated by the product of two matrices of rank m×r and r×n, with r<min(m,n), respectively. Hence, we can store only (m+n)×r matrix elements instead of m×n. The main advantage is that the algorithm is purely algebraic; hence, its formulation and implementation are integral equation kernel (Green's function) independent, also with respect to the frequency. It can be used, without an ongoing solution, in all the frequency range down to DC.

Moreover, it has been demonstrated that the ACA algorithm results in $O(N\log_2 N)$ complexity, when applied to static and electrically small electromagnetic problems.

B. NI Preconditioner

In order to improve the convergence of the iterative methods, a dedicated preconditioner was introduced. The basic idea is to exploiting the low-rank nature of matrices $\boldsymbol{P}$ and $\boldsymbol{Lp}$. In particular, the structure is hierarchically divided in blocks. Then, a sparse approximation of these matrixes ($\widehat{\boldsymbol{P}}$, $\widehat{\boldsymbol{Lp}}$) is evaluated, by considering only the interaction between basis functions in the same or in near blocks. The (very) sparse matrix

$$\widehat{\mathbf{M}} = \begin{bmatrix} \left(\widehat{\mathbf{Z}} + j\omega\widehat{\mathbf{L_p}}\right) & \mathbf{A}\widehat{\mathbf{P}} \\ -\mathbf{A}^{t} & j\omega\mathbf{U} \end{bmatrix} \qquad (8)$$

is built, and it approximated inverse is evaluated, by means of ILUT algorithm. This approximated LU factorization, called Near Interaction (NI)-Preconditioner, is used as preconditioner for the iterative solver.

Even if the PEEC method is much more stable at low frequencies respect to the canonical EFIE Method of Moments, its solution matrix can still exhibits quite high condition numbers. Indeed, since charge and current order of magnitude can be significantly different, the coefficients describing the interactions are several orders of magnitude different, following a $1/\omega$ rule. This may result in slow convergence of iterative solvers at low frequency. This limitation can be mitigated by an opportune scaling, as presented in [6].

IV. RESULTS

This section reports some examples of analysis with the described formulation. Further results will be shown at the time of conference. The first structure is the hollow cylinder shown in Fig. 1, fed by a loop, a common solution used to excite a structure resembling a fuselage in case of lightning analysis. The cylinder consists of a 4 mm sheet of material with σ = 20000 S/m, length equal to 20 m, and diameter of 4 m. The left and right caps of the cylinder and the loop are perfectly conducting. The two feeding terminals are located in the middle of the horizontal side of the loop.

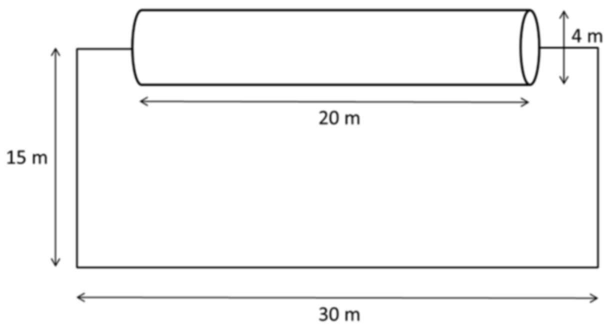

Fig. 1. Hollow cylinder with feeding loop. A port is placed at the middle of the loop.

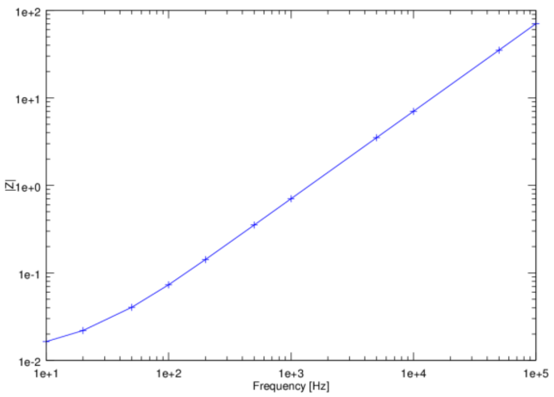

Fig. 2. Impedance of the structure of Fig. 1.

Fig. 2 shows the impedance of the structure, including the feeding loop. The same structure was analyzed in absence of the feeding loop, by direct injection of the electric current on the caps. The resulting impedance is shown in Fig. 3, showing the influence of the feeding loop. The possibility of inject and collect an electric current can allow lightning analysis in situation more close to the real one.

Fig. 4 shows two pictures of a mock-up realized and measured by Labinal Power System. Fig. 5 shown the impedance of a loop realized by connecting to the ground network a side of a wire installed on the mock-up, while Fig. 6 shows the contribution on the loop impedance of the wire and of the ground network. Also in this case, this is possible by injecting an exciting current directly on the ground network terminals.

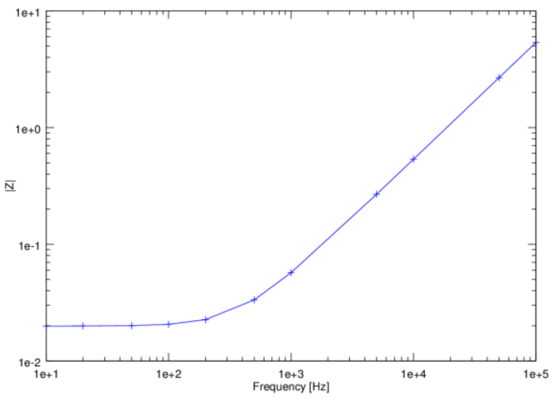

Fig. 3. Impedance of the hollow cylinder of Fig. 1 in absence of the feeding loop. Electric current is injected and collected from the two sides.

Fig. 4. Mock-up structure used in validation.

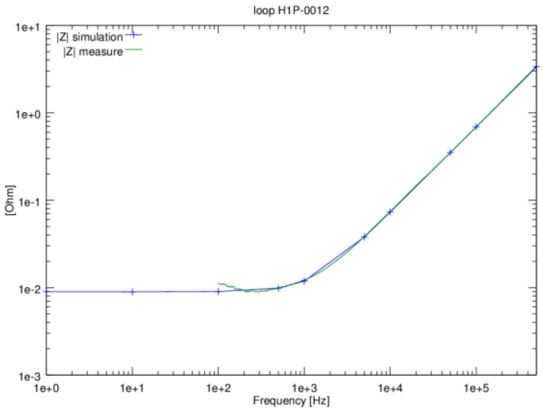

Fig. 5. Comparison between measured and simulated impedance of loop obtained short-circuiting a wire in the mock-up.

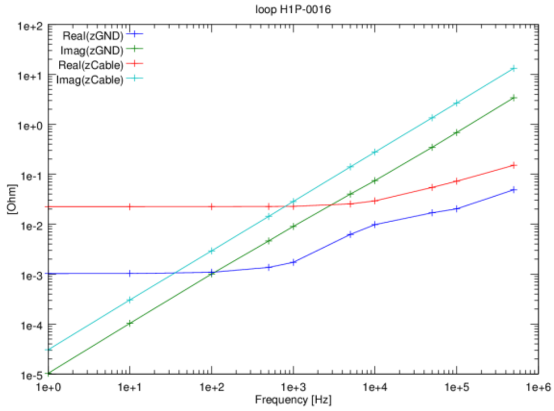

Fig. 6. Impedance of a wire and the relative the ground connection point on the mock-up.

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