Broadband full-wave frequency domain PEEC solver using effective scaling and preconditioning for SIPI models

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Introduction

Large PEEC models may require iterative solvers because direct solvers are unfeasible.

The iterative solution may be slow to converge because of the ill-conditioning of the matrix, especially at low frequency

Based on Modified Nodal Analysis, aim of this work is twofold:

1) resort to a pertinent **scaling** of sub-matrices which improves the conditioning of the global left hand side matrix;

2) use of a **multiscale-based preconditioner**

The efficiency of the resulting **scaled-preconditioned PEEC solver** is terms of either accuracy and number of iterations is demonstrated through its application to a relevant problem.

PEEC Notation

- $\mathbf{\Phi}$ the vector of node potentials to infinity of surface nodes;
- $\mathbf{I}$ the vector of vector of currents;
- $\mathbf{A}$ the connectivity matrix;
- $\mathbf{L}_p$ the partial inductance matrix;
- $\mathbf{P}$ the coefficient of potential matrix;
- $\mathbf{R}$ the resistance matrix of conductors;
- $\mathbf{I}_s$ the vector of current sources;
- $\mathbf{V}_s$ the vector of voltage sources induced by external fields.

PEEC Notation

Enforcing Kirchoff voltage and current laws to the PEEC circuit yields

$$\underbrace{\begin{bmatrix} \mathbf{R} + j\omega\mathbf{L}_p & \mathbf{A} \\ -\mathbf{P}\mathbf{A}^T & j\omega\mathbf{I}_d + \mathbf{P}\mathbf{Y}_{le} \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} \mathbf{I} \\ \mathbf{\Phi} \end{bmatrix} = \begin{bmatrix} -\mathbf{V}_s \\ \mathbf{P}\mathbf{I}_s \end{bmatrix}$$

Scaled Modified Nodal Analysis

$$\underbrace{\begin{bmatrix} \frac{1}{jk_0\mu_0 c_0}\mathbf{R} + \widetilde{\mathbf{L}}_p & \mathbf{A}\widetilde{\mathbf{P}} \\ -\mathbf{A}^T & \frac{jk_0}{\epsilon_0 c_0}\mathbf{Y}_{le}\widetilde{\mathbf{P}} - k_0^2\mathbf{I}_d \end{bmatrix}}_{\mathbf{M}_A} \begin{bmatrix} jk_0\mathbf{I} \\ c_0\mathbf{Q} \end{bmatrix} = \begin{bmatrix} -\frac{1}{c_0\mu_0}\mathbf{V}_s \\ jk_0\mathbf{I}_s \end{bmatrix}$$

$$\widetilde{\mathbf{P}} = \epsilon_0\mathbf{P}$$

$$\widetilde{\mathbf{L}}_p = \frac{1}{\mu_0}\mathbf{L}_p$$

Preconditioner

A typical effective way to reduce the number of iterations of iterative solvers, is to resort to preconditioning.

The construction of a robust preconditioner is not an easy task and it may bring along a number of problem-dependent parameters to tune.

Diagonal preconditioners do not capture near interactions.

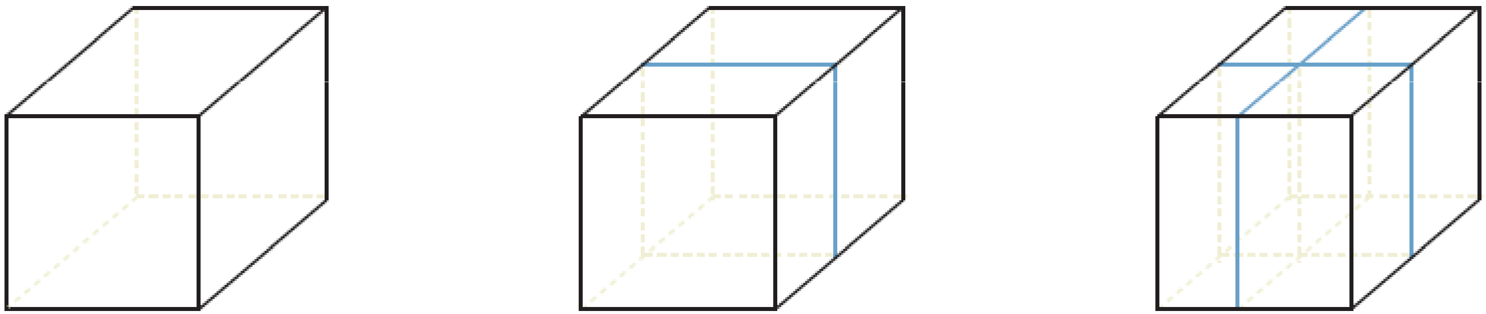
The proposed preconditioner is based on a multiscale decomposition of the domain which allows to capture near interactions without the necessity to use the time-demanding incomplete LU decomposition.

NI-Preconditioner – Domain Decomposition

The first step for the construction of the NI-Preconditioner is to perform a recursive domain subdivision of the volume containing the object:

1. The cartesian axis along which the domain (or subdomain) is largest is identified;
2. The domain (or sub-domain) is cut in correspondence of the coordinate of the average value of basis functions on the chosen cartesian axis;
3. Steps 1 and 2 are repeated recursively, for each subdomain, until reaching the final level (L) of the decomposition.

NI-Preconditioner – Domain Decomposition



(a) The box enclosing the system. (b) One level of decomposition. (c) Two levels of decomposition.

NI-Preconditioner

- Two sparse matrices $\hat{\mathbf{L}}_{\mathbf{p}}$ and $\hat{\mathbf{P}}$ are filled from $\mathbf{L}_p$ and $\mathbf{P}$, respectively, in which the only nonzero values are the elements that represent the near interactions at the final level L of the decomposition.
- The NI-Preconditioner is constructed from $\mathbf{M}_A$ as

$$\hat{\mathbf{M}}_{\mathbf{A}} = \begin{bmatrix} \frac{1}{jk_0\mu_0 c_0}\mathbf{R} + \frac{1}{\mu_0}\hat{\mathbf{L}}_{\mathbf{p}} & \epsilon_0 \mathbf{A}\hat{\mathbf{P}} \\ -\mathbf{A}^T & \frac{jk_0}{c_0}\mathbf{Y}_{le}\hat{\mathbf{P}} - k_0^2\mathbf{I}_d \end{bmatrix}$$

$$\widetilde{\mathbf{P}} = \epsilon_0 \mathbf{P}$$

$$\widetilde{\mathbf{L}}_p = \frac{1}{\mu_0}\mathbf{L}_p$$

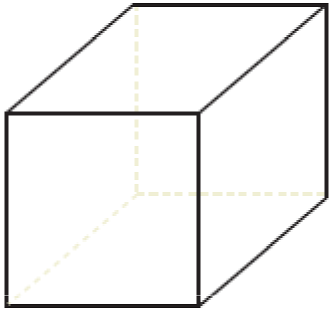
$$\underbrace{\begin{bmatrix} \frac{1}{jk_0\mu_0 c_0}\mathbf{R} + \widetilde{\mathbf{L}}_p & \mathbf{A}\widetilde{\mathbf{P}} \\ -\mathbf{A}^T & \frac{jk_0}{\epsilon_0 c_0}\mathbf{Y}_{le}\widetilde{\mathbf{P}} - k_0^2\mathbf{I}_d \end{bmatrix}}_{\mathbf{M}_A}$$

NI-Preconditioner – Sparsification

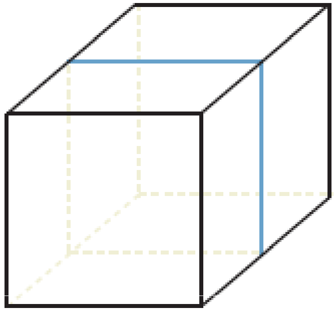
By choosing the total number of levels L, the percentage of sparsification of matrices $\hat{\mathbf{P}}$ and $\hat{\mathbf{L}}_\mathbf{p}$ can be easily controlled:

- with one level of decomposition the sparsification is about 50%;
- with two levels of decomposition the sparsification is about 75%;
- with three levels of decomposition the sparsification is about 87.5% and so on.

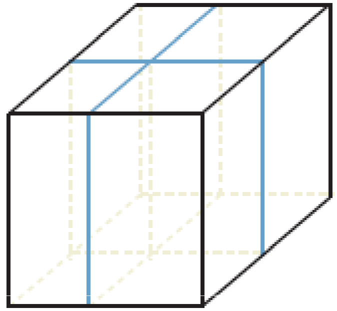
NI-Preconditioner – Sparsification



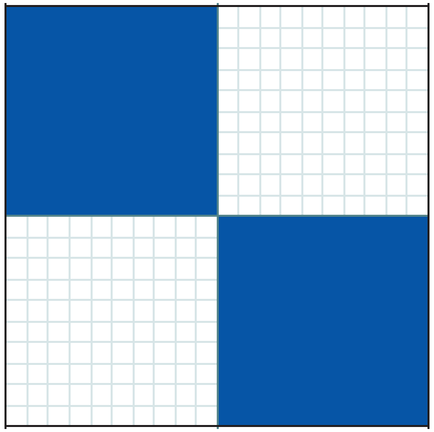
(a) The box enclosing the system.



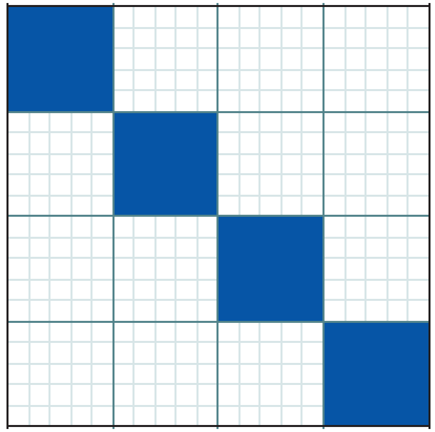
(b) One level of decomposition.



(c) Two levels of decomposition.

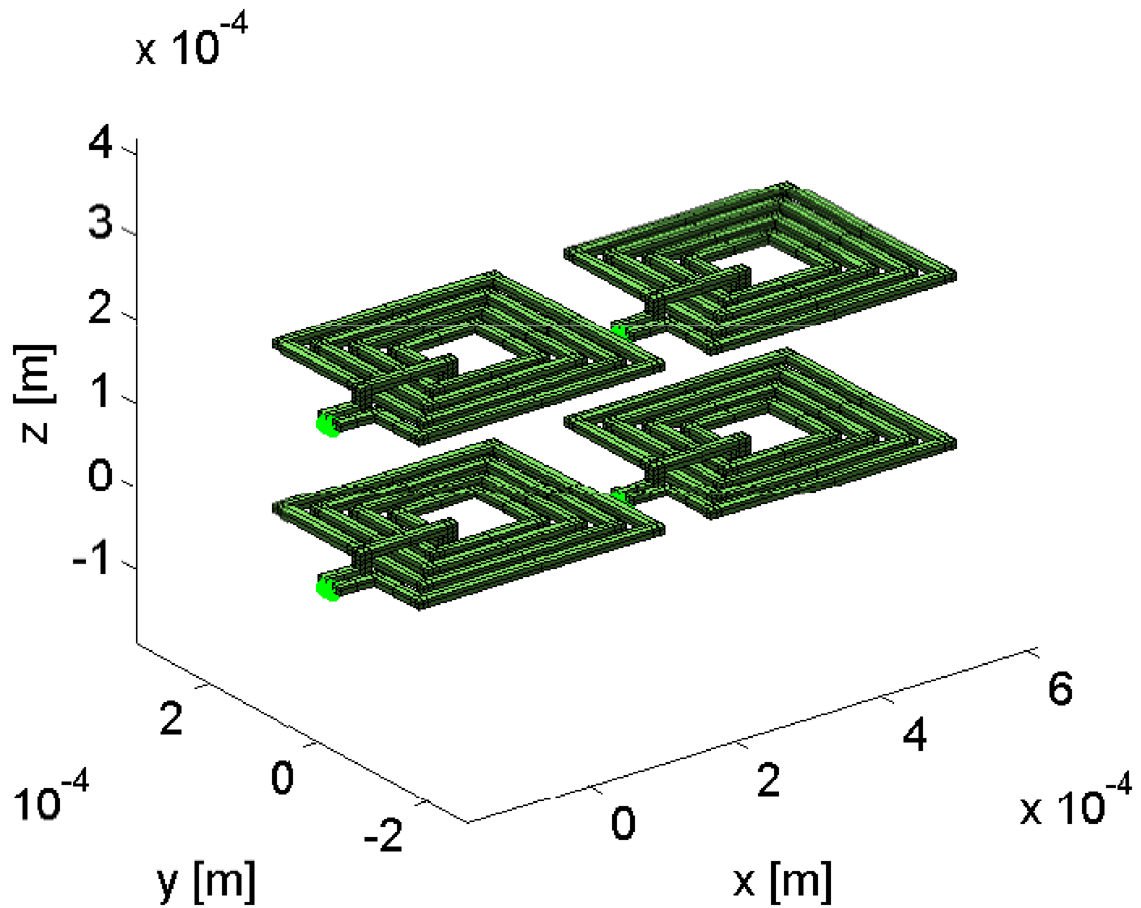


(a) One level of decomposition.

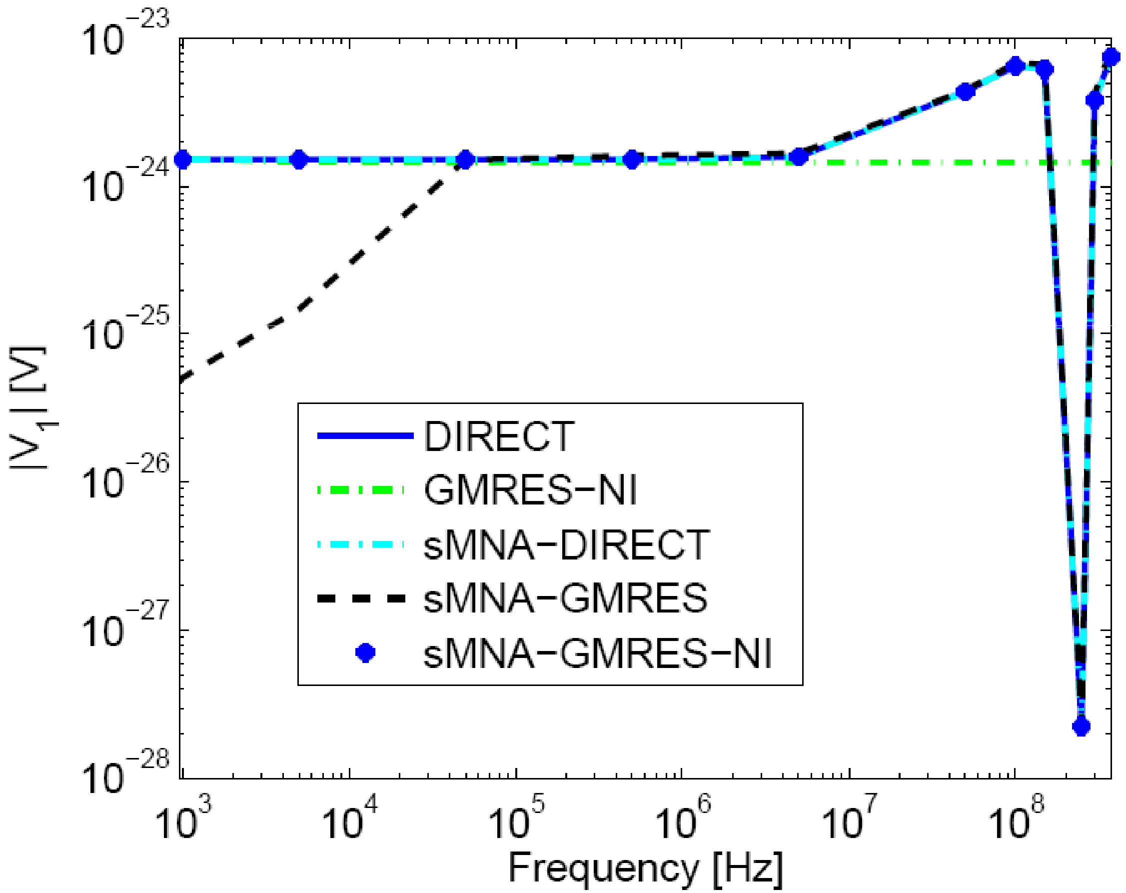


(b) Two levels of decomposition.

Low Power 3D Link example



Low Power 3D Link example



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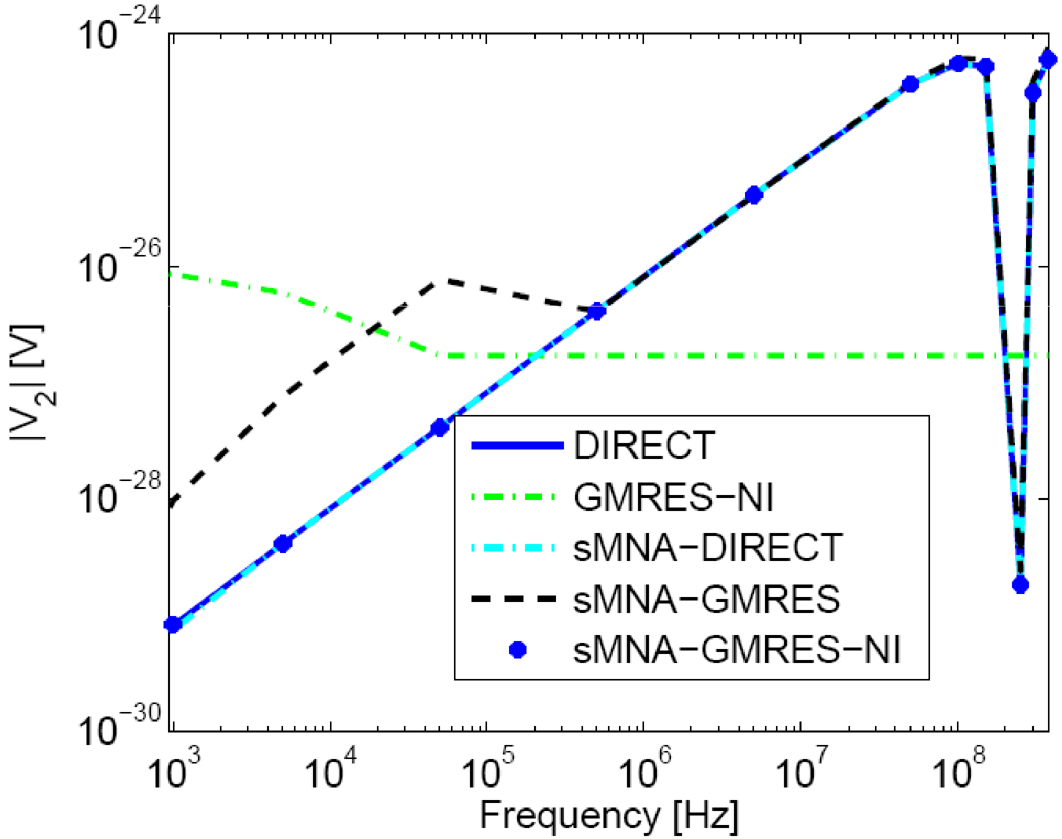

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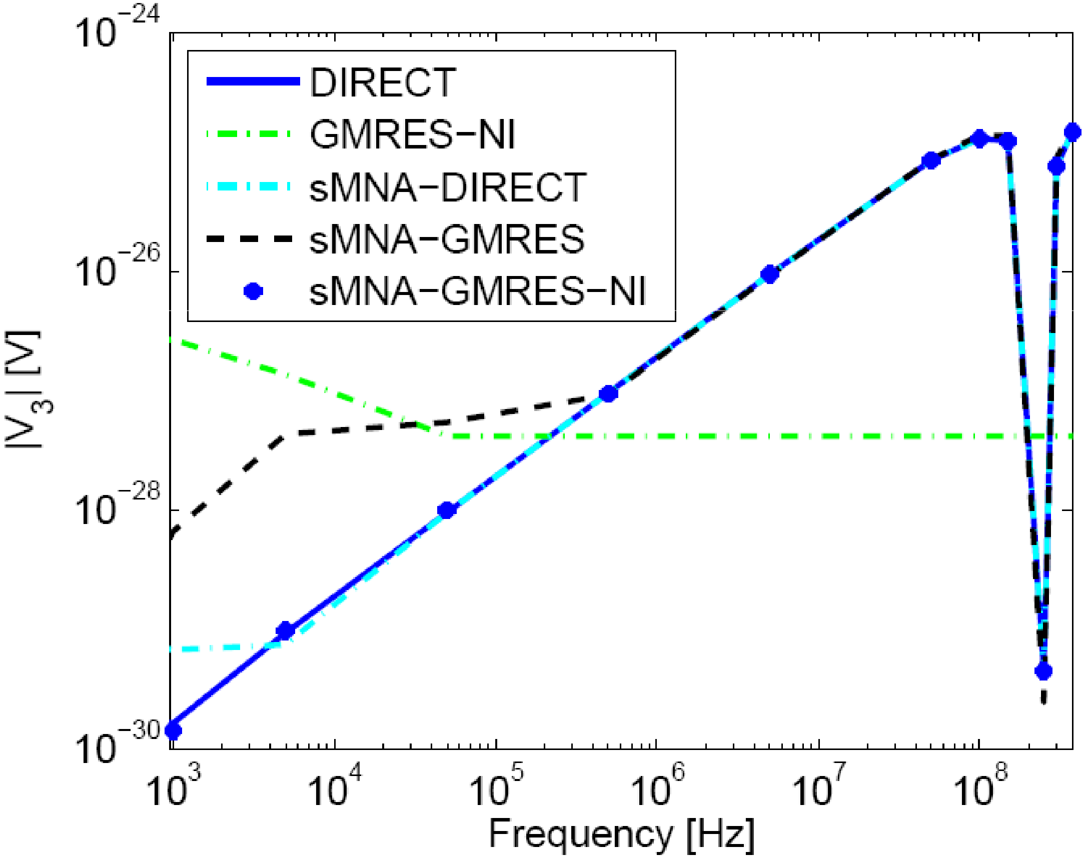

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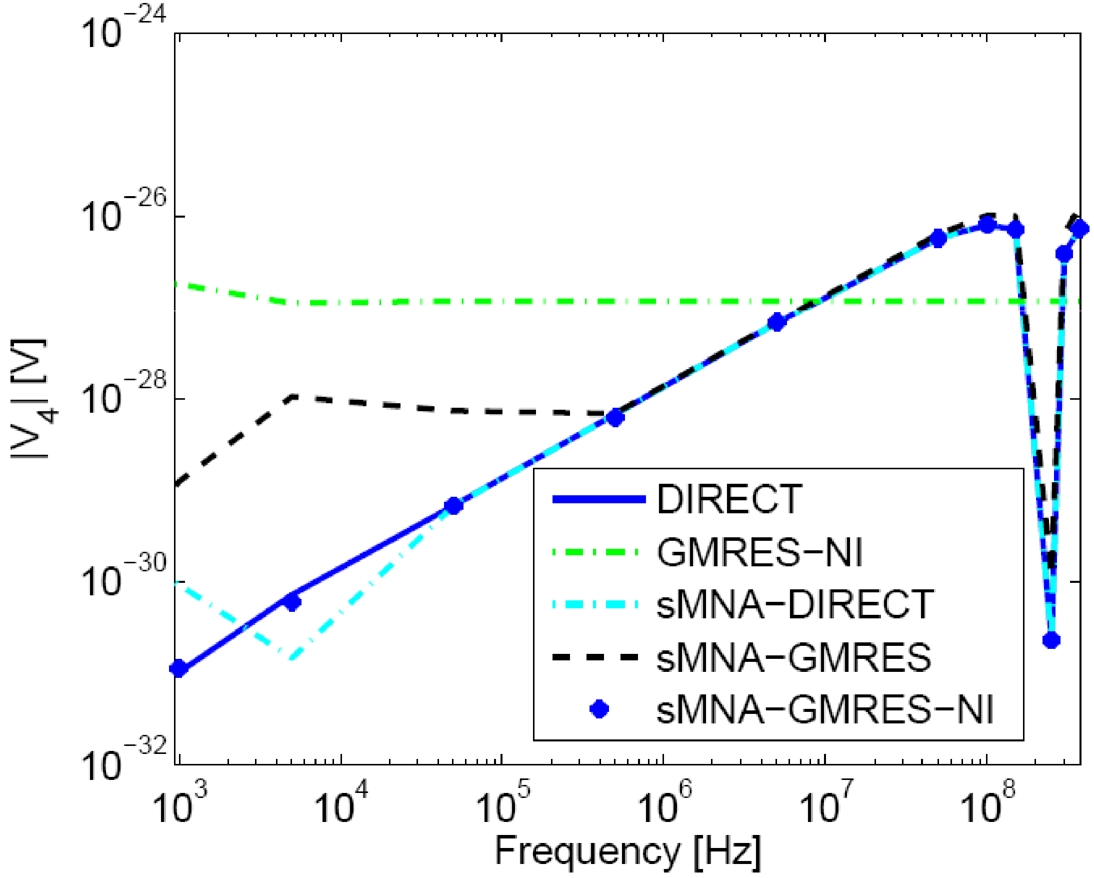

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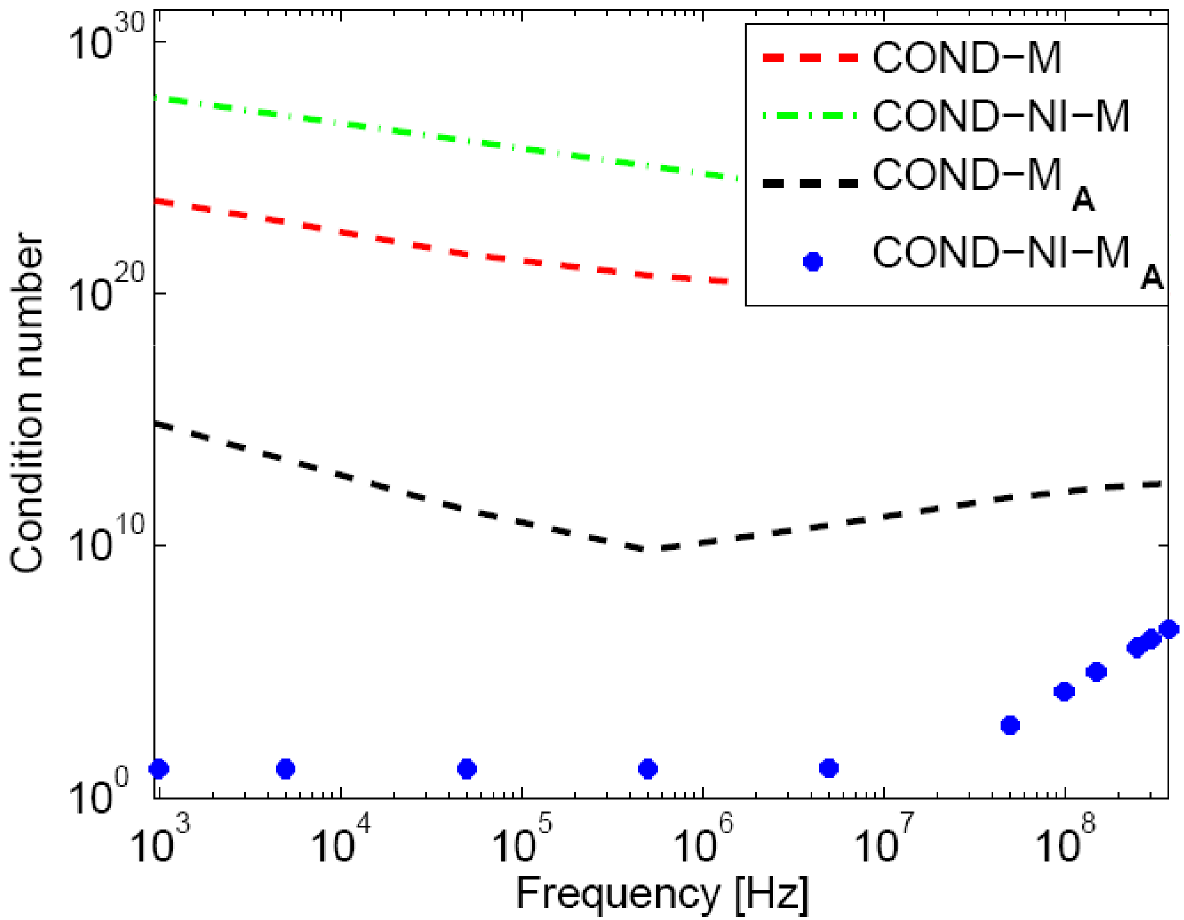
Low Power 3D Link example



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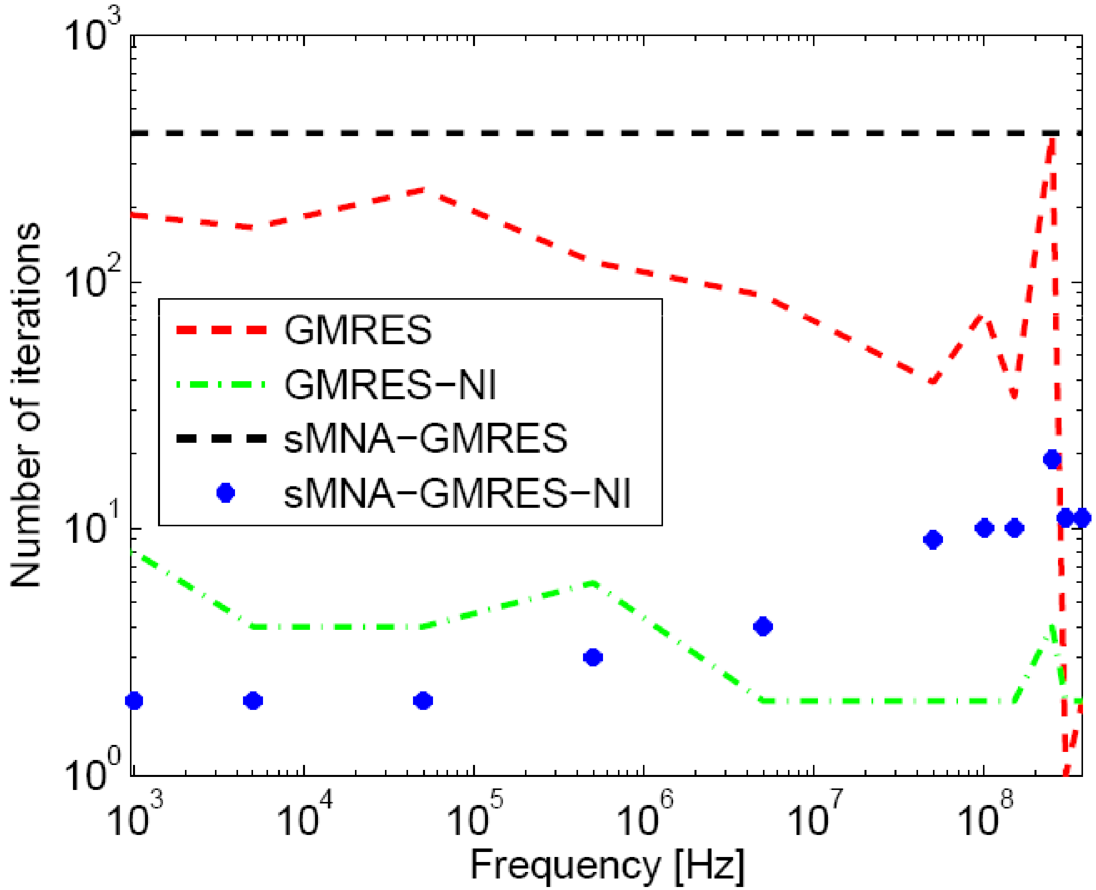

Conclusions

Broadband iterative solution of PEEC models may be an issue.

A scaled-preconditioned PEEC solver is proposed which keeps the number of iterations minimal while preserving the accuracy.
It is based on

1) a pertinent **scaling** of sub-matrices which improves the conditioning of the global left hand side matrix;

2) a **multiscale-based preconditioner**

The efficiency of the resulting **scaled-preconditioned PEEC solver** is terms of either accuracy and number of iterations is demonstrated through its application to a relevant problem.